FR n° 2962 du CNRS
Mathématiques des Pays de Loire
Région PAYS DE LA LOIRE
Géanpyl — Géométrie et Analyse en Pays de Loire

GÉANPYL : Colloque de clôture

Nantes, 30 juin — 4 juillet 2014

Roland Abuaf (Imperial College, London) *Holomorphically symplectic categories*

Homological Mirror Symmetry has been proposed by Kontsevich 20 years ago and is still a driving conjecture in mathematical string theory. In the special case of hyperkaehler manifolds, homological mirror symmetry predicts a number of autoequivalences of the derived categories which do not come from automorphisms of the complex structure.
These autoequivalences have been proved to exist for many hyperkaehler manifolds. However, the very limited number of hyperkaehler varieties we know makes it difficult to understand if these observations are significant or not. Hence, it seems very important to have more examples of derived categories of such varieties. Or perhaps not so much derived categories of hyperkaehler manifolds as such, but at least examples of triangulated categories which closely look like them.
In this talk, I will introduce the notion of *holomorphically symplectic categories*. I will explain some of their basic properties and show how Kuznetov's theory of categorical crepant resolutions of singularities enables me to produce new examples of such categories. I will in particular focus on an exotic 4 dimensional example for which I can prove the existence of $\mathbb{P}^2$-twists à la Huybrechts-Thomas. This confirms some implications of homological mirror symmetry in the non-commutative setting.

Luca Avena (WIAS, Berlin) *On some random forests with determinantal roots*

We study a certain probability measure on the set of spanning rooted oriented forests of a given finite graph. We prove that the set of roots sampled from this measure is a determinantal process and we derive several further results relating this measure to the Markov process associated with the starting graph, to the spectrum of its generator and to hitting times of subsets of the graph. This is joint work with Alexandre Gaudillière.

Antoine Ayache (Université de Lille I) *Uniformly convergent estimators for Multifractional Hurst functions*

Multifractional processes have been introduced in the 90's in order to overcome some limitations of the well-known Fractional Brownian Motion (FBM) due to the constancy in time of its Hurst parameter H; in their context, this parameter becomes a Hölder continuous function $H(\cdot)$. Global and local path roughness of a multifractional process are determined by values of $H(\cdot)$; therefore, several authors have been interested in their statistical estimation, starting from discrete variations of the process. Because of the complex dependence structure of variations, showing consistency of estimators is a tricky problem which often requires hard computations. The main goal of our talk, is to introduce in the setting of the non-anticipative moving average Linear Multifractional Stable Motion (LMSM) with a stability parameter $\alpha \in (1, 2]$, a new strategy for dealing with the latter problem. In contrast with the previous strategies, this new one, does not require to look for sharp estimates of covariances related to variations; roughly speaking, it consists in expressing them in such a way that they become independent up to negligible remainders. Thanks to it, we obtain:

1. for each compact interval I, a strongly consistent estimator of $\min_{t\in I} H(t)$;
2. more importantly, a strongly consistent estimator of the whole function $H(\cdot)$, which converges in the sense of the uniform norm.

This is a joint work with Julien Hamonier.

Hermine Biermé (Université de Poitiers) *Self-similar random fields, Chentsov's representation and applications*

For a σ-finite measure m on $(\mathbb{R}^k, \mathcal{B}(\mathbb{R}^k))$, one can define a random measure (or noise) as a set-indexed stochastic process with respect to Borelians with m-finite measure. An analogous of the central limit theorem for set-indexed sum is obtained under weak dependence conditions, by considering a self-similar measure m, to get a random Gaussian measure at the limit. Chentsov's type representation of self-similar random fields given by Takenaka allows to deduce an invariance principle for fractional Brownian fields. Replacing the Gaussian measure by a Poisson one, we may define the fractional Poisson field that we compare to the fractional Brownian field. These results are illustrated by several studies in medical imaging.

Vincent Borrelli (Université de Lyon I) *À la découverte des fractales lisses*

En 1954, un mathématicien hors catégorie, tout à la fois athlète, prix Nobel d'économie, héros de cinéma, inventeur de jeux et surtout cerveau d'exception fait une découverte fracassante. Il met en lumière l'existence de surfaces à la fois grumeleuse et lisse ; une propriété inconcevable pour l'esprit humain. Soixante ans plus tard, la première visualisation informatique d'une telle surface a permis de résoudre le paradoxe de leur mystérieuse géométrie tout en débouchant sur la découverte d'une nouvelle classe d'objets mathématiques : les fractales lisses.

Pierre Calka (Université de Rouen) *Probabilités et géométrie : mosaïques aléatoires et autres modèles*

La géométrie aléatoire est l'étude d'objets issus de la géomètres euclidienne dont le comportement relève du hasard. Elle est née avec l'énoncé du problème de l'aiguille de Buffon en 1733 mais n'a réellement pris son essor que depuis un demi-siècle. Son développement est lié d'une part à un intérêt purement mathématique qui provient de sa filiation avec la géométrie convexe et la géométrie intégrale et d'autre part au fait qu'elle intervient naturellement dans la modélisation de phénomènes observés en sciences expérimentales comme la physique des matériaux, la géologie ou l'astrophysique. Dans cet expose, plusieurs modèles couramment étudiés seront introduits et en particulier les mosaïques aléatoires. Nous ferons un tour d'horizon des résultats connus, des exemples d'application ainsi que de quelques questions ouvertes.

Serge Cohen (Université de Toulouse III) *A central limit theorem for the sample autocorrelations of a Lévy driven continuous time moving average process*

In this article we consider Lévy driven continuous time moving average processes observed on a lattice, which are stationary time series. We show asymptotic normality of the sample mean, the sample autocovariances and the sample autocorrelations. A comparison with the classical setting of discrete moving average time series shows that in the last case a correction term should be added to the classical Bartlett formula that yields the asymptotic variance. An application to the asymptotic normality of the estimator of the Hurst exponent of fractional Lévy processes is also deduced from these results.

Goulwen Fichou (Université de Rennes I) *Regulous functions*

We study the ring of rational functions admitting a continuous extension to the real affine space. We give a geometrical characterization of prime ideals of this ring in terms of their zero-locus and relate them to euclidean closed Zariski-constructible sets.

Joel Fine (Université libre de Bruxelles) *A gauge theoretic approach to Einstein's equations in dimension 4*

I will survey joint work with Dmitri Panov and Kirill Krasnov. Our basic idea is to rewrite Einstein's equations in the language of gauge theory. The fundamental objects are no longer Riemannian metrics, but instead certain connections over a 4-manifold. A connection A determines a metric g_A via its curvature, analogous to the relationship between the electromagnetic potential and the field in Maxwell's theory. The total volume of (M, g_A) is an action for the theory whose critical points are connections giving Einstein metrics. A particularly nice feature of this story is that there is also a symplectic 6-manifold associated to each connection A, which is a 2-sphere bundle Z over the 4-manifold M. The symplectic geometry of Z and Riemannian geometry of (M, g_A) are closely linked.

After covering all of this, I will explain some open problems coming out of this picture. On the one hand it suggests a way to answer certain classification questions six-dimensional symplectic geometry. On the other, it potentially provides a way to find new examples of Einstein manifolds.

Damien Gayet (Université de Grenoble Alpes) *Betti numbers of random nodal sets*

In a compact Riemannian manifold, we take at random a function f in the space spanned by the eigenfunctions of the Laplacian, with eigenvalues less than L. I will give a bound from above, as L grows to infinity, of the average of the i-th Betti number of the vanishing locus of f (the nodal sets), in terms of L and of random matrices. This is a joint work with Jean-Yves Welschinger.

Jacques Istas (Université de Grenoble Alpes) *Estimating self-similarity through complex variations*

We estimate the self-similarity index of a H-sssi process through complex variations. The advantage of the complex variations is that they do not require existence of moments and can therefore be used for infinite variance processes.

Krzysztof Kurdyka (Université de Savoie) *Stratified-algebraic vector bundles*

We investigate stratified-algebraic vector bundles on a real algebraic variety X. A stratification of X is a finite collection of pairwise disjoint, Zariski locally closed subvarieties whose union is X. A topological vector bundle ξ on X is called a stratified-algebraic vector bundle if, roughly speaking, there exists a stratification $\mathcal{S}$ of X such that the restriction of ξ to each stratum S in $\mathcal{S}$ is an algebraic vector bundle on S. In particular, every algebraic vector bundle on X is stratified-algebraic. It turns out that stratified-algebraic vector bundles have many surprising properties, which distinguish them from algebraic and topological vector bundles. This is joint work with W. Kucharz.

Ronan Le Guével (Université de Rennes II) *Statistical test for some multistable processes*

Multistable processes, that is, processes which are, at each "time", tangent to a stable process, but where the index of stability varies along the path, have been recently introduced as models for phenomena where the intensity of jumps is non constant. After the definition of various constructions of these processes, we will present how to estimate two parameters of interest of theses models, and we will explain how we can obtain a statistical test in order to decide if a data set comes from a stable process or a multistable one.

Julien Poisat (Leiden University) *An annealed charged polymer*

In 1991 Kantor and Kardar introduced a charged polymer model. The polymer is modeled by a random walk carrying random charges and weighted by a certain energy function. Each self-intersection of the random walk contributes to this energy by a positive or negative term, depending on whether the signs of the intersecting monomers agree or not. We consider the annealed version of this model, when the walk is one-dimensional. We prove the existence of the annealed free energy and give a spectral characterization. According to the value of the inverse temperature and the charge bias, the polymer can be in a ballistic or a non-ballistic phase. This is joint work in progress with Francesco Caravenna, Frank den Hollander and Nicolas Pétrélis.

Pierre Py (Université de Strasbourg) *Holomorphic maps between ball quotients*

A ball quotient is a compact or finite volume complex manifold whose universal cover is isomorphic to the unit ball in $\mathbb{C}^N$. It is in general difficult to construct examples of surjective holomorphic maps between ball quotients, besides covering maps. A few examples have been constructed and studied by Mostow, Toledo and Deraux. In this talk I will explain how to construct a few more examples. This relies on the work of Couwenberg, Heckman and Looijenga.

Arvind Singh (Université Paris XI) *Reinforced random walk*

The reinforced random walk is a simple model for an interacting process that has a tendency to come back to those sites that have been visited already. In dependence of the reinforcement mechanism there is a large variety of possible behaviors for the random walk (recurrence, localization...). In this talk, I will speak mostly about the 1-dimensional case for which sharp results are available.

Yukinobu Toda (Kavli IPMU, Tokyo) *S-duality conjecture for Calabi-Yau 3-folds*

In this talk, I give the transformation formula of Donaldson-Thomas invariants counting two dimensional torsion sheaves on Calabi-Yau 3-folds under flops. The error term is described by the Dedekind eta function and the Jacobi theta function, and the result gives evidence of a 3-fold version of Vafa-Witten's S-duality conjecture. As an application, I show that the generating series of Donaldson-Thomas invariants on the local projective plane with any positive rank is described in terms of modular forms and theta type series for indefinite lattices.

Olivier Zindy (Université de Paris VI) *Log-correlated Gaussian fields: study of the Gibbs measure*

Gaussian fields with logarithmically decaying correlations, such as branching Brownian motion and the $2D$ Gaussian free field, are conjectured to form a new universality class of extreme value statistics (notably in the work of Carpentier and Ledoussal and Fyodorov and Bouchaud). This class is the borderline case between the class of IID random variables, and models where correlations start to affect the statistics. In this talk, I will describe a general approach based on rigorous works in spin glass theory to describe features of the Gibbs measure of these Gaussian fields. I will focus on a model de fined on the periodic interval $[0; 1]$. At low temperature, we show that the normalized covariance of two points sampled from the Gibbs measure is either 0 or 1. This is used to prove that the joint distribution of the Gibbs weights converges in a suitable sense to that of a Poisson-Dirichlet variable. This is joint work with Louis-Pierre Arguin.

[*27 juin 2014*]