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[Nantes, Matpyl] Activité de printemps : Fibrés vectoriels sur les courbes.

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Jean Leray (Nantes) Matpyl Workshop
Dates:
03/03/2008 - 09:00 - 08/03/2008 - 12:00

Matpyl Spring activity on vector bundles overs curves

We will start on Monday at 9h. Registration 8h30!

This page is an overview of the topics that will be treated by the activity. Names behind the lectures indicate the distribution of the talks to be given by the participants.

There will be 6 lectures of 45 minutes a day.

Current program (PDF)

Monday

AM : *Lecture 1 [Chloé Grégoire]*

- Definition of a vector bundle over a smooth curve X defined over an algebraically closed field k (of any characteristic)
- Definition of (semi-)stability of a vector bundle
- Basic examples : if X is the projective line, Grothendieck's theorem (with proof: [HL] Thm 1.3.1), Lazarsfeld's evaluation bundles
- Elementary properties: stable implies simple; the category of semi-stable vector bundles of fixed slope is abelian.

References [HL],[Se],[LeP]

Lecture 2 [Arvid Perego]

- The fundamental group $\pi_1(X)$ if $k = C$.
- Vector bundles : E_ρ coming from representations ρ of the fundamental group .
- Theorem of Weil (see e.g. [Se] page 46) (without proof)
- Theorem of Narasimhan-Seshadri (without proof). Corollary: E and F stable implies $E \otimes F$ semi-stable

References [Se]

Lecture 3 [Heinrich Hartmann]

- The Jordan-Hölder filtration of a semi-stable bundle
- The Harder-Narasimhan filtration of a bundle; existence and uniqueness the Harder-Narasimhan filtration is stable under base field extension (with proof) see [HL] Theorem 1.3.7

References: [LeP] and [HL]

PM : *Lecture 4 [Yashonidhi Pandey]*

- The algebraic fundamental group $\pi_1^{alg}(X)$
- Bundles E_ρ coming from continuous representations ρ of the algebraic fundamental group; these are the étale trivial bundles ([LaSt])
- The absolute/relative Frobenius morphism of the curve X in positive characteristic
- Bundles coming from representations are fixed under the Frobenius ([LaSt] Satz 1.4).

References: for general facts on the algebraic fundamental group see e.g. chapter 9 by A. Mézard in [BLR], on the relative/absolute Frobenius see e.g [Ray] section 4.

Lecture 5/6 [Christian Lehn & Markus Zowislok]

- Definition of (E, ∇) bundle with connection ∇ and p-curvature of ∇ in char $p > 0$
- Cartier's theorem on descent under Frobenius
- The generalized Verschiebung on vector bundles and some of its properties (without proof)
- Frobenius-destabilized vector bundles, definition of strongly semi-stability
- E and F strongly semi-stable implies $E \otimes F$ strongly semi-stable (with sketch of proof [RR]).

Reference: [K] section 5, [Ray] section 4, [LP], [RR]

Tuesday : Principal G-bundles

AM : *Lecture 1 [Manfred Lehn]*

- Review of basics in representation theory of algebraic groups (in any characteristic)
- Definition of semi-simple/reductive algebraic groups, maximal torus, Borel subgroup, parabolic subgroup, root system, weight lattice, Weyl group

Reference: [Spr]

Lecture 2 [Tanja Becker]

- Definition of principal G-bundle E_G over a smooth curve X with G reductive.
- Comparaison local triviality in étale topology and local trivialility in Zariski topology.
- Extension of structure group $G \to H$, associated fibre bundle $E_G(Y)$ for an action of G on Y, important case: Y is a linear representation of G
- Reduction of the structure group of E_G to $H \subset G$.
- Automorphism group $Aut(E_G)$ of a principal G-bundle.
- One has $Z(G) \subset Aut(E_G)$.
- Examples: $G = GL(r)$, $SL(r), SO(r), O(r)$.

References: [So], [Ra]

Lecture 3 [Heinrich Hartmann]

- Description of G-bundles E_G as tannakian functor
- Topological classification of G-bundle
- Various definitions of semi-stability of E_G: degree of $T^|$, characters of parabolic subgroups $P \subset G$
- For $G = GL(r)$ one recovers semi-stability for vector bundles.

References: [Ra], [So1], [N]

PM : *Lecture 4 [Olivier Serman]*

- Semi-stable G-bundles with fixed topological type form a bounded family (in any characteristic)

Reference: [Beh] (8.2.6), [HN]

Lecture 5 [Jochen Heinloth]

Existence and uniqueness of canonical reduction of E_G (with sketch of proof…)

References: mainly [B]; see also [BH], [AB]

Lecture 6 [Jochen Heinloth]

- Behrend's conjecture ([B] conjecture 7.6)
- Its implications (rationality of canonical reduction), low-height representations, and a counter-example to Behrend's conjecture for $G = G_2$ and $p = 2$.

References: [B], [IMP], [He2]

Wednesday: moduli spaces

AM : *Lecture 1 [Alessandra Sarti]*

- functors of points, universal family, scheme (co-) representing a functor, existence of Grothendieck Quot scheme
- notions de quotients (catégorique, bon, géométrique)

References: [HL] Section 2.2, section 4; [Do], [LeP]

Lecture 2 [Alessandra Sarti]

- Introduction to GIT. Les critères utiles de semi-stabilité

References: [LeP]

Lecture 3 [Samuel Boissière]

- Construction GIT de M(r,d)

References: [LeP]

PM *Lecture 4 [Samuel Boissière]*

- Construction GIT de M_G + propriétés

References: [BLS]

Lecture 5 [Olivier Serman]

- Semi-stable reduction theorem(s)
- Show that $M_X(G)$ is proper in characteristic zero or if characteristic p large: do first the vector bundle case, then go to G-bundles.

References: [Lan], [HL] section 2B for the vector bundle case; [BP], [F], [He1] for G-bundles

Lecture 6 [Etienne Mann]

- Principal G-bundles over elliptic curves
- Sketch of proof that the moduli space $M_X(G)$ is isomorphic to a weighted projective space over an elliptic curve X

[FMW], [Las]

Thursday: Conformal blocks and Verlinde formula

AM : *Lecture 1 [Timo Schürg]*

- Introduction to moduli *stacks* of G-bundles.
- Definition of algebraic stack, the stack of G-bundles is algebraic, smooth. Its dimension.

[Go], [So1]

Lectures 2/3 [Manfred Lehn]

- Uniformization of G-bundles, loop spaces

References: [So1], [BL], [LS2], [F2]

PM : *Lecture 4 [Christoph Sorger]*

- Infinite Grassmannians, line bundles over the stack of G-bundles

References: [So1],[So3],[BL], [LS2], [F2]

Lecture 5 [Manfred Lehn]

- Representations of affine Lie algebras,
- space of (co)-vacua (conformal block)
- Virasoro algebras, Sugawara construction.

[So2], [SU]

Lecture 6 [Christian Lehn]

- Isomorphism between space of conformal blocks and space of generalized theta functions

[BL], [LS2]

Friday: projective connections, WZW and Hitchin

AM : *Lectures 1/2/3 [Christian Pauly]*

- Constructions of projective connections on the spaces of covacua by WZW and by Hitchin, comparison

PM : *Lectures 4/5/6 [Christoph Sorger]*

- Fusions rings and the Verlinde formula

[Be], [So2]

Saturday: Level-Rank duality

AM : *Lecture 1/2 [Rémy Oudompheng]*

- Level-Rank (or strange) duality of theta-functions

Lecture 3 [everybody]

- Open questions, conjectures, ideas, what to do next …

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